**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?

🡪Answer: Z= (X- μ)/sigma

Z= (50-/8

= 0.625

P= 1-stats.norm.cdf(0.625)

= 0.267

1. 0.3875
2. 0.2676
3. 0.5
4. 0.6987
5. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
6. More employees at the processing center are older than 44 than between 38 and 44.

🡪Answer: False [because in normal distribution most of the data is at the mean and median of the data ]

1. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

🡪Answer: TRUE

1. If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

🡪Answer: For 2X1 the mean and the variance is multiplied by factor 2, whereas for

X1 + X2  the mean is the sum of two means and the variance is the sum of the two variance.

🡪Incase X1  + X2  it got negative kurtosis as the mean is distributed widely.

1. Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.

🡪Answer: E

The probability of random variable taking a value between them is 0.99

P(a<x<b)=0.99,mean=100,var=20^2,std =20

P(0.5)= stats.norm.ppf(0.5)=-2.5758

P(99.5)=stats.norm.ppf(0.995)=2.5758

Z=(x-μ)/std

X=std\*z+μ

=20\*-2.5758+100=48.5

=20\*2.5758+100= 151.5

1. 90.5, 105.9
2. 80.2, 119.8
3. 22, 78
4. 48.5, 151.5
5. 90.1, 109.9
6. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
7. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.
8. Specify the 5th percentile of profit (in Rupees) for the company
9. Which of the two divisions has a larger probability of making a loss in a given year?

Answer: let X be the sum of two variables

E(X)= [45\*(profit1+profit2)

= [45\*(5+7)]=540 million rupees

SD[X] =SD \*[profit1+profit2] =45\*

= 45 = 225 million rupees

A)from empirical rules,

= µ ± 2σ = 540±2\*225= [90,990]

B) µ-1.5\*σ = 540-1.5\*225= 202.5

C) for division 1 [Z1]=(X-µ)/σ = (0-5)/3=-1.6666=0.048

For division 2 [Z2]= (X-µ)/σ = (0-7)/4 =-1.75 =0.040

The division 1 probability is higher so, the first division has larger probability to make loss.